The Robot Doctor

Episode 105: Robot Motion

Common Core Standards:

- Circle Circumference
- Speed, Distance and Time:
  - Linear and Angular Velocity and the relationship between them
- Basic Trigonometry:
  - Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Review:

Orientation is the direction the robot is facing, measured counter-clockwise from the x-axis (typically).
Distance is equal to the amount of rotation of a wheel times the wheels radius

$$S = \theta_{wheel} r_{wheel}$$

Linear and angular velocity measure the speed in a line, and of rotation, respectively.

$$speed = \frac{distance}{time} = linear\ velocity$$

$$\begin{matrix} speed\ of \\ rotation \end{matrix} = \frac{\begin{matrix} angular \\ displacement \end{matrix}}{time} = angular\ velocity$$

We can equate linear and angular velocities by dividing our distance equation by time

$$\theta/t = \frac{S/t}{r}$$

$$\omega = \frac{v}{r} \text{ or } \omega \cdot \mathrm{r} = v$$

Using these concepts we can determine our new position based on our starting position (x_0,y_0) and our initial heading θ_0.

$$x_t = x_0 + \omega_{w,avg} r_w t \cos(\theta_r)$$

$$y_t = y_0 + \omega_{w,avg} r_w t \sin(\theta_r)$$

$$\omega_{w,avg} = \frac{\omega_{right} + \omega_{left}}{2}$$

$$\theta_{r,t} = \theta_{r,0} + \frac{r_w}{L}(\omega_{right} - \omega_{left})t$$

These equations are only valid for small time steps since θ is changing

For slow moving robots 0.1-1 second may be sufficient

For high speed aircraft 0.1-1 millisecond may be required

Challenge Questions

Imagine you have a robot that is 50cm wide, with 10cm radius wheels. The robot starts out at (0,0) with an initial orientation of $\frac{\pi}{4}$.

1) If the robot drives both wheels at a constant speed of 1 radian per second for 10 seconds -what is the robot's final position and orientation?

2) Now what If the robot runs the right wheel at 1 radian per second and the left wheel at 1.5 radians per second. What is the robot's orientation after 1 sec of motion?